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COMMENT

The falsity of a conjecture concerning the percolation threshold

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Abstract. A recent conjecture relating the critical concentration for bond percolation and a lattice Green function is shown to be false.

Recently Sahimi *et al* (1983) have conjectured that the critical concentration $p_c^{(d)}$ for nearest-neighbour bond percolation on the d-dimensional hypercubic lattice is exactly equal to the corresponding lattice Green function $G_0^{(d)}$. The latter is given by

$$G_0^{(d)} = \frac{1}{N}\sum_q \frac{1}{2d(1-\gamma_q)}, \tag{1}$$

where the sum is taken over the first Brillouin zone consisting of N points and

$$\gamma_q = (\cos q_1 a + \cos q_2 a + \ldots + \cos q_d a)/d, \tag{2}$$

where q_n is the nth component of the d-dimensional lattice vector q. Here we show that this conjecture is false.

To do this we simply compare the expansion of the two quantities in powers of d^{-1} for large d. From the work of Gaunt and Ruskin (1978) we have

$$p_c^{(d)} = (1/2d)[1 + 1/2d + 7/8d^2 + \ldots]. \tag{3}$$

The expansion of $G_0^{(d)}$ takes the form

$$G_0^{(d)} = \frac{1}{2dN}\sum_q \sum_{n=0}^{\infty} (\gamma_q)^{2n}. \tag{4}$$

Keeping terms up to $n = 2$ we have

$$G_0^{(d)} = \frac{1}{2d}\left(1 + \frac{1}{2d} + \frac{6d-3}{8d^3} + \mathrm{O}(d^{-3})\right) = \frac{1}{2d}\left(1 + \frac{1}{2d} + \frac{3}{4d^2} + \mathrm{O}(d^{-3})\right). \tag{5a, b}$$

Comparing equations (3) and (5) we obtain

$$p_c^{(d)} - G_0^{(d)} = 1/16d^3 + \mathrm{O}(d^{-4}).$$

Thus $p_c^{(d)}$ and $G_0^{(d)}$ are definitely not exactly equal in general dimension. For $d = 2$, in fact, $G_0^{(d)}$ is infinite, whereas $p_c^{(d)}$ is finite.

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